

Non-time-orthogonality and Tests of Special Relativity

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Abstract

An intriguing, and possibly significant, anomalous signal in the Brillet and Hall experiment is contrasted with a simple first order test of special relativity subsequently performed to discount that signal as spurious. Analysis of the non-time-orthogonal nature of the rotating earth frame leads to the conclusion that the latter test needed second order accuracy in order to detect the effect sought, and hence was not sufficient to discount the potential cause of the anomalous signal. The analysis also explains the results found in Sagnac type experiments wherein different media were placed in the path of the light beams.

1 Introduction

In 1981 Aspen[1] showed that a persistent non-null test signal recorded by Brillet and Hall[2] in the most modern Michelson-Morley type experiment performed to date would correspond to a test apparatus velocity of 363 m/sec. As the earth surface speed at the test location is approximately 355 m/sec, it was suggested that perhaps the rotating earth's surface velocity is somehow subtly different from the solar and galactic orbital velocities, which had long since been shown to yield null signals. To date there has been no other Michelson-Morley type test with sufficient accuracy to detect a possible effect from the earth surface velocity.

In 1985, Byl et al[3] performed a first order test of special relativity which ostensibly showed no effect from the earth surface velocity, and led to the conclusion that the Brillet and Hall signal was erroneous. The Byl et al test was simple, clever, and based on an analysis that included certain, seemingly reasonable, kinematic and constitutive assumptions.

In this article we first review this test/analysis, then note that underlying assumptions are not in accord with that derived from non-time-orthogonal (NTO) analysis of rotating frames. It is then shown that if the NTO derivation result is correct, the effect of the earth surface velocity would be second, not first, order, and hence undetectable by the Byl et al experiment. This re-opens the question of the meaning and validity of the Brillet and Hall anomalous signal.

2 “First Order” Test of Special Relativity

2.1 The Experiment

Byl, Sanderse, and van der Kamp split a laser beam into two, directed one beam through air and one through water, and then combined the resultant beams to yield an interference pattern. According to their analysis (summarized below), the existence of an ether (privileged frame) would cause one of the beams to be slowed more than the other when both were aligned with the direction of absolute velocity. This would result in relative phase shifting and a movement of the fringes as the apparatus was turned.

2.2 The Analysis

Byl et al considered two cases: i) a direct Galilean transformation between the earth and ether frames with no ether drag, and ii) a Fresnel ether drag effect. We note that conclusions for each were similar and so only review the first case.

Assume a Galilean universe with light speed c relative to the ether. The test apparatus moves towards the right with speed v . The top half of the original beam is directed toward the right through air while the bottom half is directed toward the right through water. Both travel a distance D where the bottom beam emerges into air, and are at that point combined to form interference fringes.

The travel time for the top beam is

$$t_1 = \frac{D}{(c - v)}. \quad (1)$$

The travel time for the bottom beam is

$$t_2 = \frac{D}{u_G} = \frac{Dn}{(c - v)} \quad (2)$$

where u_G is the presumed Galilean speed of light in water, and n is the index of refraction of water.

The time difference is

$$T_a = t_2 - t_1 = \frac{D(n - 1)}{(c - v)}. \quad (3)$$

When the apparatus is rotated 180° the difference in travel time becomes

$$T_b = \frac{D(n - 1)}{(c + v)}. \quad (4)$$

The change in time difference is thus

$$\Delta T = T_b - T_a = \frac{2D(n - 1)vc}{c^2 - v^2}. \quad (5)$$

The displacement length of one wave to the other is then $c\Delta T$, leading to a displacement in M wavelengths, or fringes, of

$$M = \frac{c\Delta T}{\lambda} = \frac{2D(n - 1)}{\lambda} \frac{vc}{c^2 - v^2} \quad (6)$$

or

$$M = \frac{2D(n - 1)}{\lambda} \frac{v}{c} + O[(v/c)^3] \quad (7)$$

Hence, the experiment was considered to have first order accuracy in v/c , be more sensitive than the (second order) Michelson-Morley experiment, and be capable of readily detecting any effect from the earth surface velocity.

With the given apparatus, accuracy was deemed to be at least 20 times that needed to check the earth surface speed hypothesis. Repeated testing showed no detectable motion of the fringes.

3 Index of Refraction and NTO Frames

3.1 NTO Frames Theory

Klauber[4]·[5] has analyzed the NTO frame metric obtained when one makes a straightforward transformation from the lab to a rotating frame such as that of a relativistically spinning disk. Unlike other researchers[6] he has not assumed that it is then necessary to transform to locally time orthogonal frames. Instead he considered what phenomena would result if one proceeded using the NTO frame metric as a physically valid representation of the rotating frame.

Klauber found time dilation and mass-energy dependence on tangential speed ωr that is identical to the predictions of special relativity and the test data from numerous cyclotron experiments. He further found resolutions of several well known, and other not so well known, paradoxes inherent in the traditional analytical treatment of rotating frames.

However, he also found some behavior that may seem somewhat unusual from a traditional relativistic standpoint, and which is difficult to verify experimentally. In particular, it was found that the physical[7]·[8]

(i.e., measured with standard instruments, not merely generalized coordinate value) velocity of light in vacuum in the circumferential direction on a rotating (NTO) frame is non-invariant, non-isotropic, and equals[9]

$$u_{light,vacuum} = \frac{\pm c - r\omega}{\sqrt{1 - (r\omega)^2/c^2}} = \frac{\pm c - v}{\sqrt{1 - v^2/c^2}}, \quad (8)$$

where ω is the angular velocity, r is the radial distance from the center of rotation, $v = \omega r$, and the sign before c depends on the propagation direction of the light ray.

This relationship leads to the prediction of a signal due to the earth surface speed v precisely like that found by Brillat and Hall[10]. For bodies in gravitational orbit (8) does not hold, and $u_{light} = \pm c$. This is because such bodies are in free fall, and are essentially inertial, Lorentzian, time orthogonal (TO) frames. They are not subject to the idiosyncrasies of non-time-orthogonality, so their orbital speed would not result in a non-null Michelson-Morley signal.

The most relevant of Klauber's findings for the present purposes is the generalization of (8) to an arbitrary velocity (other than for light). This, the transformation of physical tangential (circumferential) velocities between inertial and rotating frames, is[11]

$$u = \frac{U - r\omega}{\sqrt{1 - (r\omega)^2/c^2}} \quad (9)$$

where lower case represents the rotating frame and upper case the inertial frame in which the rotation axis is fixed.

3.2 Speed of Light in Water

As NTO frames have unique properties, it is not appropriate to simply assume that a beam of light in such a frame passing through a medium fixed in that frame would have speed $(c \pm v)/n$, as was presumed in (2) and (4). However, given what we know about NTO frames and Lorentz frames, we can determine what a rotating frame observer would see for a beam of light passing through a medium fixed in a Lorentz frame having the same instantaneous velocity as that observer. It may then seem reasonable to assume that this speed is the proper one to use in calculations such as those of section 2.2. Further support for this assumption may be found in the Appendix, which provides an alternative derivation for the results of this section.

Consider three frames: the rotating frame k , the non-rotating frame K in which the axis of rotation is fixed, and a Lorentz frame K_1 , having velocity $v = \omega r$ (relative to K) in the direction of the instantaneous velocity of a point fixed in k . K_1 is sometimes called a "tangent" or a "co-moving" frame. Consider a medium such as water fixed in K_1 with a light beam passing through it in the direction of \mathbf{v} . As measured in K_1 , a Lorentz frame, the speed of that beam is c/n where n is the index of refraction of the medium.

K and K_1 are both Lorentz frames and the transformation law for velocities between such frames tells us that the light beam in question as seen from K has speed

$$U = \frac{c/n + v}{1 + \frac{(c/n)v}{c^2}} = \frac{c}{n} + v - \frac{v}{n^2} + O[1/c]. \quad (10)$$

Transformation of U to the rotating frame is done using (9) whereupon we find the circumferential speed of the light beam in a transparent medium is seen from the rotating frame to be

$$u = \frac{c}{n} - \frac{v}{n^2} + O[1/c] = \frac{c}{n} \left(1 - \frac{v}{cn} + O[1/c^2] \right). \quad (11)$$

4 Fringe Analysis Revisited

Repeating the analysis method of section 2.2, we find the travel time for the top beam (in air) from (8) as

$$t_1 = \frac{D\sqrt{1 - v^2/c^2}}{(c - v)} = \frac{D}{c - v} + O[1/c^3] = \frac{D}{c} \left(1 + \frac{v}{c} \right) + O[1/c^3]. \quad (12)$$

The travel time for the bottom beam with the light in water velocity of (11) is

$$t_2 = \frac{D}{u} = \frac{D}{(c/n)(1 - v/cn + O[1/c^2])} = \frac{Dn}{c} \left(1 + \frac{v}{cn}\right) + O[1/c^3]. \quad (13)$$

The time difference where, importantly, terms in v/c^2 cancel is

$$T_a = t_2 - t_1 = \frac{D}{c} (n - 1) + O[1/c^3]. \quad (14)$$

Ignoring higher order terms, one finds this result is independent of v , and so, through second order, it will also equal T_b , the time difference when the apparatus is turned 180° . Therefore the fringe shift change

$$M = \frac{c\Delta T}{\lambda_{air}} = \frac{c(T_b - T_a)}{\lambda_{air}} = O[1/c^2] \quad (15)$$

is second, not first order. (Wavelength in air is used in (15) since the standing wave interference pattern occurs in air after the bottom beam has emerged from the water.)

Thus, no matter which way the apparatus is turned, there will be no first order phase shift between water and air light beams. Hence, if k is the earth frame, there will be no observable change in fringe location in the experiment of section 2.

5 Sagnac Experiments with Transparent Media

Post[12] summarized results of several Sagnac type experiments, some of which analyzed the effect of fixing transparent media to the rotating apparatus in the paths of the oppositely directed light beams. In repeated testing, no interference fringe change was observed between the light beams in air and light beams in other media configurations. Change was, however, observed when the media was stationary while the apparatus rotated[13].

The Galilean analysis leading to (7) is in conflict with these results, as it would predict a first order fringe shift between tests with no media and those with media fixed to the rotating apparatus. A purely Lorentzian analysis, while predicting no such shift for different media fixed to the apparatus, would also predict no fringe change for variations solely in angular velocity. That such change does in indeed occur is an indisputable experimental fact. Unlike either the Galilean or the Lorentz frame analysis, the NTO frame analysis predicts fringe changes in accord with all of the cited experiments.

5.1 Sagnac and Rotating Media

Tests with media fixed to the rotating apparatus are governed in NTO analysis by (15), which represents the fringe shift difference between waves passing through air and through a medium along the same path in a rotating frame. To first order this is zero regardless of direction of velocity v . Hence, both the counterclockwise and clockwise beams have no first order retardation difference, resulting in no observable fringe shift prediction, and agreement with experiment.

5.2 Sagnac and Non-rotating Media

As noted, Sagnac tests in which a transparent medium is fixed in the lab exhibit fringe shifting between the no media and media cases. We analyze this as follows.

Analogously with (12), the times of travel as measured on the rotating frame for the clockwise (cw) and counterclockwise (ccw) light beams in air around a circumference D using NTO analysis [see (8)] where $v = \omega r$ are

$$t_{cw,air} = \frac{D}{u_{cw,air}} = \frac{D\sqrt{1 - v^2/c^2}}{(c - v)} = \frac{D}{c - v} + O[1/c^3] \cong \frac{D}{c} \left(1 + \frac{v}{c}\right) \quad (16)$$

and

$$t_{ccw,air} = \frac{D}{u_{ccw,air}} = \frac{D\sqrt{1 - v^2/c^2}}{(c + v)} \cong \frac{D}{c} \left(1 - \frac{v}{c}\right). \quad (17)$$

Times for cw and ccw light to travel the same path D for glass stationary while the apparatus rotates are found by noting that light speed in the glass media in the lab is c/n . Then from (9) with $U = c/n$ we have

$$t_{cw,glass} = \frac{D}{u_{cw,glass}} = \frac{D\sqrt{1-v^2/c^2}}{c/n-v} = \frac{Dn}{c} \left(\frac{1}{1-nv/c} \right) + O[1/c^3] \cong \frac{Dn}{c} \left(1 + \frac{nv}{c} \right) \quad (18)$$

and

$$t_{ccw,glass} = \frac{D}{u_{ccw,glass}} = \frac{D\sqrt{1-v^2/c^2}}{c/n+v} \cong \frac{Dn}{c} \left(1 - \frac{nv}{c} \right). \quad (19)$$

The travel time differences between the glass and air beams for each of the cw and ccw directions are then

$$\Delta t_{cw} = t_{cw,glass} - t_{cw,air} \cong \frac{D}{c} \left((n-1) + (n^2-1)\frac{v}{c} \right) \quad (20)$$

and

$$\Delta t_{ccw} = t_{ccw,glass} - t_{ccw,air} \cong \frac{D}{c} \left((n-1) - (n^2-1)\frac{v}{c} \right). \quad (21)$$

The difference in reduction of arrival times[14] between the cw and ccw beams is

$$\Delta t_{cw/ccw} = \Delta t_{cw} - \Delta t_{ccw} \cong 2D(n^2-1)\frac{v}{c^2}, \quad (22)$$

so the fringe shift location change between the air and stationary glass tests is

$$M = \frac{c\Delta t_{cw/ccw}}{\lambda_{air}} \cong \frac{2D}{\lambda_{air}}(n^2-1)\frac{v}{c}. \quad (23)$$

Hence NTO analysis of stationary media placed in the Sagnac experiment light paths predicts a first order fringe shift increase in agreement with the test of Dufour and Prunier[15] noted by Post[16].

6 Summary and Conclusions

The persistent non-null signal found in the Brillet and Hall experiment, the most modern and accurate Michelson-Morley type experiment to date, appears to correlate with the earth surface velocity. Non-time-orthogonal analysis of the rotating frame of the earth predicts this signal and implies that it can not be deemed spurious on the basis of the subsequent first order test carried out by Byl et al.

Sagnac type experiments carried out with various transparent media in the path of the light beams yield results that are in harmony with the predictions of NTO frame analysis, but are at variance with Lorentzian and Galilean frame analyses.

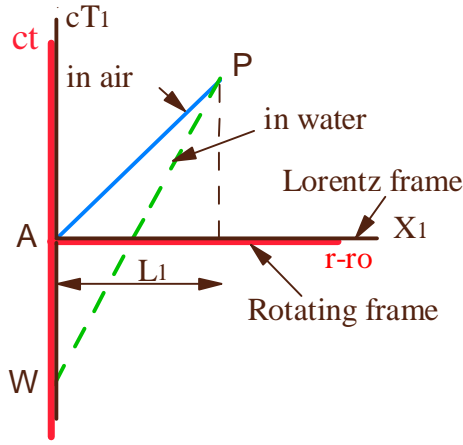
NTO analysis is ultimately consonant with the theory of relativity. Due to the idiosyncratic nature of NTO frames, however, it makes some predictions that do not seem traditionally relativistic.[17]

Appendix: Light Speed in Rotating Medium

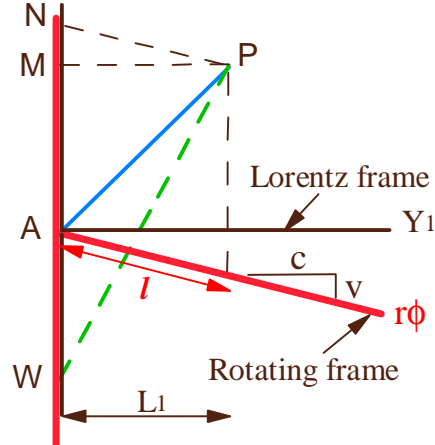
This appendix provides an alternative derivation to (11). Some knowledge of NTO frames (see reference [4]) is assumed.

Figure 1 depicts the radial direction and time axes at a point fixed in the rotating frame k for both the rotating frame and a Lorentz frame K_1 having velocity equal to the tangential velocity of that point. The X_1 axis of K_1 is aligned with the radial axis of k . Event A has coordinates $(ct=0, r=r_o, \phi=0, z=0)$ in k and $(cT_1=0, X_1=0, Y_1=0, Z_1=0)$ in K_1 . All distances and times are small. Both the r axis in k and the X_1 axis in K_1 are orthogonal to their respective time axes. Only first order effects are considered, so any time dilation or Lorentz contraction effects are ignored. Hence, the coordinate time t in k is, for present purposes, effectively equal to the time on local standard clocks in k , as well as time T_1 in K_1 .

Each frame carries a tube of water aligned in the radial direction as well as standard clocks and measuring rods to determine both the speed of a light beam in water and that of a parallel light beam in air. (“Air” is considered equivalent to “vacuum” here.) The light beam in air travels spacetime path AP from event A to



**Fig 1. Light in Two Media,
Radial Direction vs Time**



**Fig 2. Circumferential
Direction vs Time**

The NTO frame treatment of the aforementioned reference analyzes identical events, but due to the NTO nature of the frame, measured quantities such as time, distance, and light speed between those events differ from that of TO frames. We assume a similar stance with regard to the light beam passing through water. That is, there appears to be no constitutive reason why the path WP of a light beam in water in the Lorentz frame K_1 should differ from that of a light beam in water in the rotating frame k . The emitting event W and the constructive interference event P should be the same.

The issue then is to determine the speed of the light beam in water as measured in k . The first step in doing this is to note that the time of event P differs in the two frames. From Figure 2 it is seen to equal to the value on the time axis of event M in K_1 , and to that of event N in k . We know the speed of light in water in the Lorentz frame K_1 is

$$V_{water,K1} = \frac{c}{n} = \frac{L_1}{\Delta t_{WM}}. \quad (24)$$

The speed of the same light beam as seen in the rotating frame k is

$$v_{water,k} = \frac{l}{\Delta t_{WN}} \cong \frac{L_1}{\Delta t_{WM} + \Delta t_{MN}}. \quad (25)$$

where the RHS results because to first order $l \cong L_1$. From the slope $-v/c = -\omega g/c$ of NP, we find

$$c\Delta t_{MN} = L_1 \frac{v}{c}. \quad (26)$$

Using this in (25) along with (24), we find

$$v_{water,k} \cong \frac{L_1}{\Delta t_{WM}} \left(1 - \frac{L_1}{\Delta t_{WM}} \frac{v}{c^2} \right) = \frac{c}{n} \left(1 - \frac{v}{cn} \right), \quad (27)$$

the same relationship (where approximately equal means to zeroth order in $1/c$) as (11).

References

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- [2] A. Brillet and J. L. Hall, “Improved laser test of the isotropy of space,” *Phys. Rev. Lett.*, **42**(9), 549-552 (1979).
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- [6] For example, Adler, R., Bazin, M., and Schiffer, M., *Introduction to General Relativity* (McGraw-Hill, New York, 1975), 2nd ed., pp. 121-122.
- [7] All quantities (such as the speed of light) used in all equations in this article are physical values, i.e., they equal what an experimentalist would measure with standard instruments. They are not coordinate values, which vary with the coordinate system used, but physical components, which are unique within a given frame and correspond to actual measured quantities. See ref. [8].
- [8] Robert D. Klauber, “Physical components, coordinate components, and the speed of light”, gr-qc/0105071.
- [9] Ref. [4], Section 4.2.4, pg. 425, eq. (19) modified by the time dilation factor discussed in the subsequent paragraphs therein to yield physical velocity., and pg. 430, eq. (33).
- [10] Ref. [4], Section 6. pp. 434-436.
- [11] Ref. [4], Section 4.2.4, pp. 424-425. See eq. (18) modified by the factor in the subsequent paragraph therein to yield physical velocity.
- [12] E. J. Post, “Sagnac Effect”, *Rev. Mod. Phys.*, **39**(2), pp. 475-493 (1967).
- [13] Ref. [12], pp. 477-478 summarizes the effects of media on Sagnac test fringing. Note that fringe shift change is defined as the change in fringe location with respect to the stationary interferometer fringe location [pg. 476 before eq. (1)]. It is *not* affected by the change in wavelength due to the insertion of transparent media in the light path. This is because the light waves emerge from the transparent media prior to forming a standing wave interference pattern, i.e., the interference fringing occurs in air. Any retardation that effects both waves to the same degree will have no effect on the standing wave interference pattern. That is, both traveling waves (that form the standing wave) arriving later by the same Δt is equivalent to looking at the two original waves Δt later in time. But the fringe pattern does not change with time. For example, in the non-rotating apparatus, insertion of a transparent medium in the paths of the light waves (such that both waves pass through the same length of the medium) will have no effect on the fringing. A change in location of interference fringes indicates one wave was retarded by a different amount than the other.
- [14] One notes that both light beams are slowed the same amount by the zeroth order (independent of $v = \omega r$) term in both (20) and (21). This term will have no effect on the location of the fringes. (See endnote [13].)

- [15] Dufour, A. and Prunier, F., *J. Phys. Radium*, 8th Ser., **3**, 153 (1942).
- [16] Ref. [12], pg. 478.
- [17] Ref. [4], Section 4.2, pp. 423-425.